Significant Figures

When writing a number, it is important to know how well the value of the number is known.

Examples

If I say I have 12 donuts, I usually mean I have exactly 12 donuts because donuts do not come in fractions of a unit.

Significant Figures

If I say I have 12 grams of salt, does that mean exactly 12 grams or something between 11.5 and 12.5 grams.

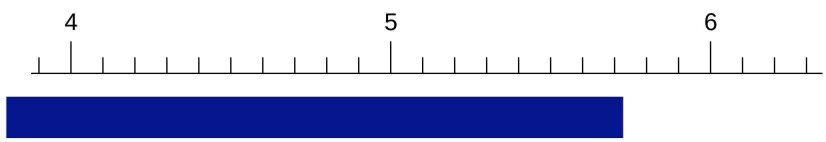
Because the mass of an object in grams may be expressed in fractions of grams, I must specify as precisely as possible what the mass is.

Significant figures indicate the precision with which a number has been determined.

Significant Figures

When expressing a value, the number of digits used indicates the number of digits that could be measured accurately.

The final digit in the value is an estimate of the least precise number.



Length = 5.73 units

Significant Figures

The manner in which a number is written indicates the number of significant figures it has.

<u>Examples</u>

4000 1 significant figure

the zeros are simply place holders and do not indicate precision of the measurement.

in scientific notation: 4×10^3

Significant Figures

4000. 4 significant figures

By placing a decimal point after the last zero, this indicates that the zeros were measured

in scientific notation: 4.000×10^3

Significant Figures

4000.000 7 significant figures

zeros following the decimal point with no other digits behind them also indicate precision of measurement, so they count as significant figures

Significant Figures

.0004 1 significant figure

for numbers less than one, zeros following the decimal point, but before the first digit, are simply place holders and do not indicate precision

in scientific notation: 4×10^{-3}

Significant Figures

.000400 3 significant figures

zeros following digits in numbers less than 1 indicate precision and are significant.

in scientific notation: 4.00×10^{-3}

Mathematical Rules for Significant Figures

Addition/Subtraction

When adding or subtracting numbers, the number of significant figures in the result is determined from the position relative to the decimal point of the least significant figure of the numbers being added or subtracted

Mathematical Rules for Significant Figures

Add 472.1, 3.192, and 5000.86

```
   472.1
     3.192
+ 5000.86
5476.152
```

Because 472.1 has only 1 sig. fig. after the decimal point, the final answer can have only 1 sig. fig. after the decimal point—the correct answer is 5476.2

Mathematical Rules for Significant Figures

Subtract 126.5419 from 8000:

8000
- 126.5419
7873.4581

Because 8000 has only 1 sig. fig. four places to the left of the decimal point, the least significant figure in the final answer must also be four places to the left of the decimal point—correct answer is 8000

Mathematical Rules for Significant Figures

Multiplication/Division

The number of significant figures in the result is determined from the number of significant figures in the least significant value used in the calculation

Mathematical Rules for Significant Figures

Multiply 88.037 by .00721

$$\begin{array}{r} 88.037 \\ \times \quad .00721 \\ \hline 0.63474677 \end{array}$$

88.037 has 5 sig. figs. and .00721 has 3 sig. figs. This limits the result to a total of three sig. figs.—the correct answer is 0.635

Mathematical Rules for Significant Figures

<u>Logarithms/Exponentiation</u>

Remember the definitions of logarithms:

$x = 10^y$ $\quad$ $\log(x) = y$

$1000 = 10^3$ $\quad$ $\log(1000) = 3$

Mathematical Rules for Significant Figures

Logarithms/Exponentiation

Logarithms have a *characteristic* and a *mantissa*:

log(87.21) = 1.9406

characteristic (1) mantissa (.9406)

The *characteristic* determines the appropriate power of ten for the result, and the *mantissa* relates to the proper value of the number

Mathematical Rules for Significant Figures

Logarithms/Exponentiation

The correct number of significant figures for a logarithmic operation is determined by the number of significant figures in the mantissa

Mathematical Rules for Significant Figures

Logarithms/Exponentiation

Example:

$\log(2.18 \times 10^{-6}) = -5.662$

the original number has 3 sig. figs., so the resulting mantissa should also have 3 sig. figs., and the result has 4 sig. figs.

-5, the characteristic, indicates the number of 0s serving as place holders

Mathematical Rules for Significant Figures

Logarithms/Exponentiation

Example:

$10^{4.5} = 3 \times 10^4$

the mantissa of 4.5 has only 1 sig. fig.

the characteristic = 4 and indicates the power of ten in the result

the result will have only 1 sig. fig.

Accuracy and Precision

Accuracy and precision have very different meaning in quantitative analysis

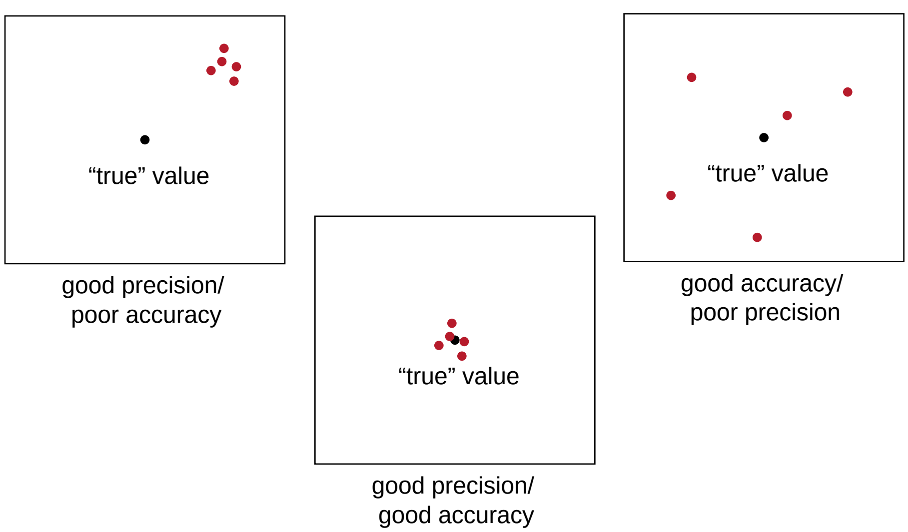
<u>Definitions</u>

Accuracy—how well a result represents the "true" value for a measurement

Precision—how well a set of measurements can be repeated with the same result

Ideally, we want both accuracy and precision in our lab work

Accuracy and Precision



Types of Experimental Errors

Determinate (systematic) errors

Determinate errors are errors that result from a flaw in the experimental procedure

- miscalibrated instrument
- operator mistakes

Determinate errors occur the same way every time an analysis is made

They can be corrected by calibrating instruments or training operators

Types of Experimental Errors

Random errors

Random errors result from the physical variability of measurements

Random errors have an equal chance of being either positive or negative

- noise in instruments
- uncertainty in making readings

Uncertainty and Errors

If we assume that determinate errors have been eliminated, the uncertainty in any quantitative result will come from random errors

Uncertainty and Errors

The uncertainty of a single measurement can be determined either:

- from the equipment used
  - a buret has an uncertainty of ± 0.02 mL
  - an analytical balance has an uncertainty of approximately ± 0.0003 g

Uncertainty and Errors

The uncertainty of a single measurement can be determined either:

- performing repeated measurements and calculating the standard deviation

when using an atomic absorption spectrometer, measure the absorbance of a sample five times

Uncertainty and Errors

Uncertainty in a result can be expressed as either absolute (e) or relative (%e) uncertainty (for now, we will use e to represent uncertainty in a quantity)

Absolute uncertainty

m = 3.125 ± 0.037 g

e = 0.037 g is uncertainty in mass

absolute uncertainty has same units as the result

Uncertainty and Errors

Relative uncertainty

Relative uncertainty is calculated by dividing the absolute uncertainty by the result and multiplying by 100%

m = 3.125 ± 0.037 g

$$\%e = \frac{0.037\text{g}}{3.125\text{g}} \times 100\% = 1.1_8\%$$

subscripted 8 indicates this digit is insignificant, but we will keep until a final answer is determined

m = 3.125 g ± 1.1_8%

relative uncertainty is always expressed as a percentage

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Addition/Subtraction

$$x_{final} = x_1 + x_2 + x_3$$

$$x_1 \pm e_1 \qquad x_2 \pm e_2 \qquad x_3 \pm e_3$$

$$e_{final} = \sqrt{e_1^2 + e_2^2 + e_3^2} \qquad x_{final} \pm e_{final}$$

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Addition/Subtraction

$$x_{final} = x_1 - x_2 + x_3$$

$$x_1 = 1.982 \pm 0.008 \quad x_2 = 0.752 \pm 0.052 \quad x_3 = 25.154 \pm .853$$

$$e_{final} = \sqrt{(.008)^2 + (.052)^2 + (.853)^2} = 0.854$$

$$x_{final} = 26.384 \pm 0.854$$

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Multiplication/Division

$$x_{final} = (x_1)(x_2)(x_3)$$

$$x_1 \pm \%e_1 \quad x_2 \pm \%e_2 \quad x_3 \pm \%e_3$$

$$e_{final} = \sqrt{\%e_1^2 + \%e_2^2 + \%e_3^2} \quad x_{final} \pm \%e_{final}$$

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Multiplication/Division

$$x_{final} = \frac{(x_1)(x_2)}{x_3}$$

$$x_1 = 2.341 \pm 1.12\% \quad x_2 = 8.34x10^{-4} \pm 3.83\% \quad x_3 = 1280 \pm 0.239\%$$

$$e_{final} = \sqrt{(1.12)^2 + (3.83)^2 + (.239)^2} = 3.99_8\%$$

$$x_{final} = 1.52_5 \pm 3.99_8\%$$

$$x_{final} = 1.52_5 \pm 0.06_1$$

Uncertainty and Errors

When performing calculation involving mixed operations (addition/subtraction and multiplication/ division)

1. Perform addition/subtraction, determine absolute error of result, and then relative error of result
2. Perform multiplication/division operation using relative errors

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Exponentiation and Logarithms

For $y = x^a$ $\qquad x \pm \%e_x$

$$\%e_y = a(\%e_x)$$

$y = x^3$ $\qquad x = 5.981 \pm 2.13\%$

$\%e_y = 3(2.13\%) = 6.39\%$

$y = 214.0 \pm 6.39\%$

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Exponentiation and Logarithms

For $y = \log(x)$ $\qquad x \pm e_x$

$$e_y = \frac{1}{\ln 10}\frac{e_x}{x}$$

$y = \log(x)$ $\quad x = 3.17 \pm 0.28$

$e_y = (.43429)\cdot(0.28)/(3.17) = 0.038_4$

$y = 0.50 \pm 0.04$

Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Exponentiation and Logarithms

For $y = \ln(x)$ $x \pm e_x$

$$e_y = \frac{e_x}{x}$$

$y = \ln(x)$ $x = 7.432 \times 10^{-5} \pm 0.917 \times 10^{-5}$

$e_y = (0.917 \times 10^{-5})/(7.432 \times 10^{-5}) = 0.123_3$

$y = -9.507 \pm 0.123$

Uncertainty and Errors

A summary of the rules for determining uncertainties and their propagation through a series of operation is given in Table 3-1