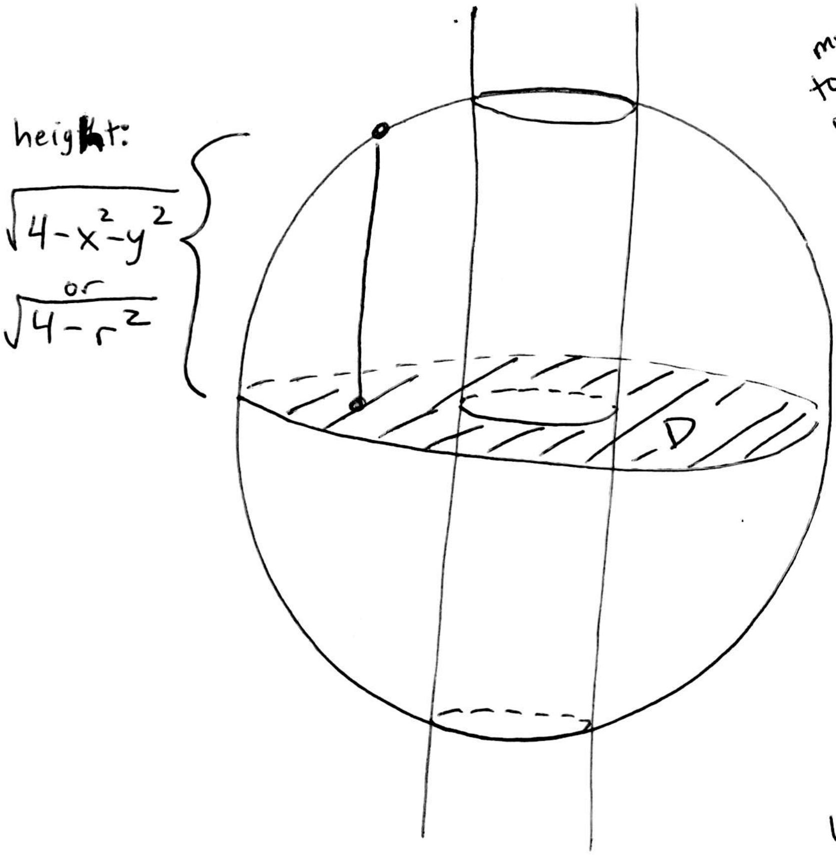
Quiz 5, Math 209, Winter 2007 **Name:** Solutions

Directions: Show ALL of your work to get credit. If you leave something out, then you may be penalized. No calculators. Good luck!

IMPORTANT: This quiz has two sides. Look at both!

1. [10 points] Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$ and outside the cylinder $x^2 + y^2 = 1$.



multiply by 2 to get volume of whole thing

volume of top half

$$V = 2\iint_D \sqrt{4-x^2-y^2}\, dA$$

outer circle radius

$$= 2\int_0^{2\pi}\int_1^2 \sqrt{4-r^2}\; r\,dr\,d\theta$$

inner circle radius

$$= 2\int_0^{2\pi}\int_3^0 \sqrt{u}\left(-\tfrac{1}{2}\,du\right)d\theta$$

$u = 4-r^2$

$du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

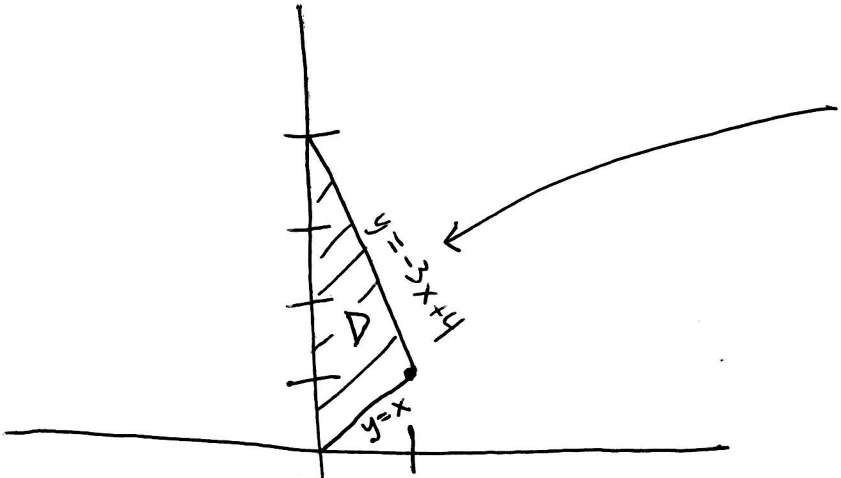
$$= 2\left(-\tfrac{1}{2}\right)\int_0^{2\pi}\int_3^0 u^{1/2}\,du\,d\theta = -\int_0^{2\pi}\left(\tfrac{2}{3}u^{3/2}\Big|_3^0\right)d\theta$$

$$= -\int_0^{2\pi} -\tfrac{2}{3}\left(3^{3/2}\right)d\theta = 2\sqrt{3}\int_0^{2\pi} d\theta = 4\pi\sqrt{3}$$

$3^{3/2} = \sqrt{3}\sqrt{3}\sqrt{3} = 3\sqrt{3}$

2. [10 points]SETUP UP BUT <u>DO NOT EVALUATE</u> AN INTEGRAL TO DO THE FOLLOWING:

Find the mass of the lamina that occupies the triangular region with vertices $(0,0)$, $(1,1)$, and $(0,4)$ [redacted] and has density function $\rho(x,y) = x$.



To find this equation:
$y = mx + b$
$b = 4.$
So, we have $y = mx + 4$,
Plug in $(x,y) = (1,1)$ to
get $1 = m + 4 \Rightarrow m = -3.$

$$\boxed{\text{mass} = \int_0^1 \int_x^{-3x+4} x \, dy \, dx}$$

$$\underbrace{\qquad\qquad}_{\iint_D \rho(x,y)\, dA}$$

<u>Another way:</u> $\displaystyle \int_0^1 \int_0^y x \, dx \, dy + \int_1^4 \int_0^{-\frac{1}{3}y + \frac{4}{3}} x \, dx \, dy$

$y = -3x + 4$
$y - 4 = -3x$
$x = -\frac{1}{3}y + \frac{4}{3}$

So, the dxdy way is harder than the dydx way.