Math 2550 - Homework # 8

Column space, nullspace, rank-nullity theorem

1. Determine whether $\vec{b}$ is in the column space of A. If so, express $\vec{b}$ as a linear combination of the columns of A.

   (a) $A = \begin{pmatrix} 1 & 3 \\ 4 & -6 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -2 \\ 10 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$

2. For each matrix A, (i) find a basis for the nullspace of A, (ii) calculate the nullity of the matrix, (iii) find a basis for the column space of A, (iv) state the rank of A, (v) verify that the rank-nullity theorem is true for A.

   (a) $A = \begin{pmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$

3. Suppose that A is a 4×5 matrix. Suppose that the nullity of A is 3. What is the rank of A ?

4. Suppose that A is a matrix where a basis for it's column space is

$$\left\{ \begin{pmatrix} 2 \\ -3 \\ 1 \\ 8 \\ 7 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 1 \\ -9 \\ 6 \end{pmatrix} \right\}$$

Also suppose that a basis for the nullspace of A contains exactly 2 vectors. How many columns does A have? Justify your answer.

5. For each set of vectors: (i) Find a subset of the given vectors that forms a basis for the space spanned by the vectors, and then (ii) express each vector that is not in the basis as a linear combination of the basis vectors.

   (a) $\vec{v}_1 = \langle 2, -1 \rangle$, $\vec{v}_2 = \langle 5, -7 \rangle$, $\vec{v}_3 = \langle 1, 1 \rangle$

   (b) $\vec{v}_1 = \langle 1, 0, 1 \rangle$, $\vec{v}_2 = \langle 0, 1, 2 \rangle$, $\vec{v}_2 = \langle 1, 1, 1 \rangle$

6. Suppose that A is a 3×3 matrix. Suppose that the nullity of A is 0. Show that every vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is in the span of the columns of A.