Def: A vector space V over a field F consists of a set "vectors" V with two operations, vector addition and scalar multiplication, such that the following hold.

1. $\vec{v}+\vec{w}$ is in V for all $\vec{v},\vec{w}$ in V.
2. $\alpha\cdot\vec{v}$ is in V for all $\vec{v}$ in V and α in F.
3. $\vec{v}+\vec{w}=\vec{w}+\vec{v}$ for all $\vec{v},\vec{w}$ in V.
4. $(\vec{v}+\vec{w})+\vec{z}=\vec{v}+(\vec{w}+\vec{z})$ for all $\vec{v},\vec{w},\vec{z}$ in V.
5. There exists a vector $\vec{0}$ in V such that $\vec{0}+\vec{v}=\vec{v}+\vec{0}$ for all $\vec{v}$ in V.
6. For each $\vec{v}$ in V there exists a vector $\vec{u}$ in V such that $\vec{u}+\vec{v}=\vec{v}+\vec{u}=\vec{0}$.
7. $1\cdot\vec{v}=\vec{v}$ for all $\vec{v}$ in V. Here 1 is the 1 element from F.
8. $(\alpha\beta)\cdot\vec{v}=\alpha(\beta\vec{v})$ for all $\vec{v}$ in V and α,β in F
9. $\alpha(\vec{v}+\vec{w})=\alpha\vec{v}+\alpha\vec{w}$ for all $\vec{v},\vec{w}$ in V and α in F
10. $(\alpha+\beta)\vec{v}=\alpha\vec{v}+\beta\vec{v}$ for all $\vec{v}$ in V and α,β in F