~~Ex: Let $a = 60$ and $b = 350$.~~
~~Then $\gcd(a,b) = \gcd(60,350) = 10$.~~
~~Note that $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = \gcd(6,35) = 1$.~~
~~This always happens!~~

Here's a more general version of the lemma

Lemma: Let $a_1, a_2, \ldots, a_n$ be integers, not all zero. Let $d = \gcd(a_1, a_2, \ldots, a_n)$. Then $\gcd\left(\frac{a_1}{d}, \frac{a_2}{d}, \ldots, \frac{a_n}{d}\right) = 1$.

In particular, for two integers $a, b \in \mathbb{Z}$ not both zero with $d = \gcd(a,b)$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

proof: Since $d = \gcd(a_1, a_2, \ldots, a_n)$ we have that $d \mid a_i$ for each i. Hence there exist integers $k_i \in \mathbb{Z}$ with $dk_i = a_i$ for $i = 1, 2, \ldots, n$.

Let $d' = \gcd\left(\frac{a_1}{d}, \frac{a_2}{d}, \ldots, \frac{a_n}{d}\right)$.

Then $d' \,\Big|\, \frac{a_i}{d}$ for all i, so there exist $l_i \in \mathbb{Z}$ with $d' l_i = \frac{a_i}{d}$ for $i = 1, 2, \ldots, n$.

Thus, $a_i = (dd')l_i$ for $i = 1, 2, \ldots, n$.

So, dd' is a positive common divisor of the a_i.

Hence $dd' \leq d$ (since d is the greatest positive common divisor of the a_i).

Thus, $d' \leq 1$ (by dividing by d).

Since d' is positive, $d' = 1$.