# Math 446 - Homework # 2

1. For the numbers $a$ and $b$ given below do the following: (i) list the positive divisors of $a$, (ii) list the positive divisors of $b$, (iii) list the positive common divisors of $a$ and $b$, (iv) calculate $\gcd(a, b)$.

   (a) $a = 12$ and $b = 24$

   (b) $a = 16$ and $b = 36$

   (c) $a = 5$ and $b = 18$

   (d) $a = 0$ and $b = 3$

2. Calculate the following:

   (a) $\gcd(12, 25, 14)$

   (b) $\gcd(30, 6, 10)$

   (c) $\gcd(12, 0, 8)$

3. Using the Euclidean algorithm, calculate the greatest common divisor of the following numbers:

   (a) 39 and 17

   (b) 2689 and 4001

   (c) 1819 and 3587

   (d) 864 and 468

4. For each problem: First determine if there are any integer solutions. If there are no solutions, explain why not. If there are solutions, then carry out these steps: (a) Use the Euclidean algorithm to find integers $x$ and $y$ that satisfy the equation, (b) give a formula for all the solutions to the equation, and (c) use your formula to find four more solutions to the equation.

   (a) $4001x + 2689y = 1$

   (b) $864x + 468y = 36$

   (c) $5x + 3y = 7$

   (d) $1819x + 3587y = 17$

(e) $10x + 105y = 101$

(f) $39x + 17y = 22$

(g) $3x + 18y = 9$

5. Suppose that $a, b, x, y$ are integers with $a$ and $b$ not both zero. Prove that $\gcd(a, b)$ divides $ax + by$.

6. Prove that no integers $x$ and $y$ exist such that $x - y = 200$ and $\gcd(x, y) = 3$.

7. Let $a$ and $b$ be integers, $a > 0$, $b > 0$, and $d = \gcd(a, b)$. Prove that $a|b$ if and only if $d = a$.

8. Let $a$ and $b$ be integers such that $\gcd(a, 4) = 2$ and $\gcd(b, 4) = 2$. Prove that $\gcd(a + b, 4) = 4$.

9. Suppose that $x, y, z$ are integers with $x \neq 0$. Prove that $x|yz$ if and only if $\left.\frac{x}{\gcd(x,y)}\right| z$.

10. Let $a, b, c$ be integers with $a \neq 0$ and $b \neq 0$. Prove that if $a|c$, $b|c$, and $\gcd(a, b) = 1$, then $ab|c$.

11. Let $a, b, c, x$ be integers with $a$ and $b$ not both zero and $x \neq 0$. Prove that if $\gcd(a, b) = 1$, $x|a$, and $x|bc$, then $x|c$.

12. Suppose that $a$ and $b$ are integers, not both zero. Suppose that there exist integers $x$ and $y$ with $ax + by = 1$. Prove that $\gcd(a, b) = 1$.

13. Show that the following is not necessarily true: If $a, b, c, x, y$ are integers and $ax + by = c$ then $\gcd(a, b) = c$.