

Math 4680 - Homework # 10

Cauchy Integral Formula

1. Evaluate the following integrals.

- (a) $\int_{\gamma} \frac{z^2}{z-1} dz$ where γ is the circle of radius 2, centered at 0, oriented counterclockwise.
- (b) $\int_{\gamma} \frac{\sin(z)}{z^2} dz$ where γ is the unit circle oriented clockwise.
- (c) $\int_{\gamma} \frac{z^2-1}{z^2+1} dz$ where γ is the circle of radius 2, centered at 0, oriented counterclockwise.
- (d) $\int_{\gamma} \frac{z^{10}+5z^3}{z^4} dz$ where γ is the square with vertices $-1-i$, $1-i$, $1+i$, $-1+i$, oriented counterclockwise.
- (e) $\int_{\gamma} \frac{1}{(z^2+z+1)^2} dz$ where γ is the circle $|z|=2$ oriented counterclockwise.
- (f) $\int_{\gamma} \frac{z}{(9+z^2)(z+i)^2} dz$ where γ is the circle $|z|=4$ oriented counterclockwise.

2. Let γ be the circle $|z|=3$ oriented counterclockwise. Define

$$g(w) = \int_{\gamma} \frac{2z^2 - z - 2}{z - w} dz$$

for all w with $|w| \neq 3$.

- (a) Show that $g(2) = 8\pi i$.
 - (b) What is the value of $g(w)$ when $|w| > 3$?
3. Suppose that f is analytic within and on a simple, closed smooth curve γ . Further suppose that $f(w) = 0$ for all w on γ . Prove that $f(z) = 0$ for all z inside of γ .