

## Math 474 - Homework # 2

### Combinatorial Counting

1. How many six-digit numbers are there? How many of them contain the digit 5?
2. Suppose that in a state, license plates have 3 letters (from A-Z) followed by 3 numbers (from 0-9) in a way that no letter or number is repeated in a single plate. Determine the number of license plates for this state.
3. (a) How many permutations of the letters  $a, b, c, d,$  and  $e$  are there?  
(b) How many begin with  $a$  and end with  $c$ ?
4. How many different messages can be sent with five dashes '-' and three dots '.' ?
5. Let  $A$  be the set of all sequences of 0's, 1's, and 2's of length 8.
  - (a) How many elements are there in  $A$ ?
  - (b) How many elements of  $A$  have exactly four 0's and four 1's?
  - (c) How many elements of  $A$  have exactly three 0's, three 1's, and two 2's?
6. (a) In how many ways can 5 math, 3 biology, 2 history, and 3 literature books be placed on a bookshelf?  
(b) In how many ways can this be done so the math books are together?
7. 5 boys and 6 girls sit in a row at random. What is the probability that the boys are together and the girls are together?
8. Six 6-sided dice are thrown. What is the probability that at least two of them show the same number?
9. From a group of 10 women and 8 men a committee consisting of 5 women and 4 men must be formed.

- (a) How many different committees are possible if there are no restrictions?
  - (b) How many different committees are possible if 2 of the men refuse to serve together?
10. Suppose that four 8-sided dice are thrown.
- (a) What is the probability that you will get exactly two 3's?
  - (b) What is the probability that you get at most two 8's?
  - (c) What is the probability that you at least three 1's?
11. Suppose that ten 6-sided dice are thrown. Calculate the probability that you will get exactly one 4, exactly six 5's, and the other three numbers are anything other than 4's or 5's.
12. Suppose a coin is tossed 5 times.
- (a) What is the probability that exactly 1 head occurs and the other four tosses are not 1's?
  - (b) What is the probability that exactly 3 heads occur?
  - (c) What is the probability that all five tosses are tails?
13. Suppose that a coin is tossed 20 times.
- (a) What is the probability that at least 2 heads occurs?
  - (b) What is the probability that at most 3 heads occurs?
14. Suppose that five numbers are selected at random from the numbers

$1, 2, 3, 4, 5, 6, 7, \dots, 19, 20$

What is the probability that the smallest number selected is larger than 6?

For example, if you selected the five numbers 6, 10, 2, 19, 20 then the smallest number 2 would not be larger than 5. However, if you selected the numbers 11, 15, 6, 9, 18 then the smallest number 6 would be larger than 5.

15. Recall from class that in the CA SuperLotto Plus you pick 5 "lucky" numbers from 1 to 47 (no repeats here and order doesn't matter) and you also pick 1 "mega" number from 1 to 27.
- (a) What is the probability that you get 2 of the 5 lucky numbers correct and the mega number correct?
  - (b) What is the probability that you get 4 of the 5 lucky numbers correct and the mega number correct?
16. Suppose from a standard 52-card deck you are dealt five cards. You only know three of the cards and they are  $2\clubsuit$ ,  $3\clubsuit$ , and  $4\clubsuit$ . You don't know the other two cards.
- (a) What are the odds that the other two cards are both clubs so you have a flush?
  - (b) What are the odds that the other two cards will give you a straight, but NOT a straight flush?
  - (c) What are the odds that the other two cards will give you a straight flush?