(B)

I'll do the splitting field for 13.4 #2.
From hw solutions for 13.4 #2 we have that $E = \mathbb{Q}(2^{1/4}, i)$ is the splitting field for $x^4 - 2$ over $\mathbb{Q}$ and $[E : \mathbb{Q}] = 8$.

Note that $f(x) = \min_{2^{1/4}, \mathbb{Q}}(x) = x^4 - 2$ and $g(x) = \min_{i, \mathbb{Q}}(x) = x^2 + 1$. If $\sigma \in \mathrm{Gal}(E/\mathbb{Q})$ then $\sigma 2^{1/4}$ a root of $x^4 - 2$. The roots of $x^4 - 2$ are $2^{1/4}, 2^{1/4} i, -2^{1/4}, -2^{1/4} i$.
Also σi is a root of x^2+1. So $\sigma i = i$ or $-i$.

Define the following maps

$$\sigma_1 = \begin{cases} 2^{1/4} \mapsto 2^{1/4} \\ i \mapsto i \end{cases} \quad \sigma_2 = \begin{cases} 2^{1/4} \mapsto 2^{1/4} i \\ i \mapsto i \end{cases} \quad \sigma_3 = \begin{cases} 2^{1/4} \mapsto -2^{1/4} \\ i \mapsto i \end{cases}$$

$$\sigma_4 = \begin{cases} 2^{1/4} \mapsto -2^{1/4} i \\ i \mapsto i \end{cases} \quad \sigma_5 = \begin{cases} 2^{1/4} \mapsto 2^{1/4} \\ i \mapsto -i \end{cases} \quad \sigma_6 = \begin{cases} 2^{1/4} \mapsto 2^{1/4} i \\ i \mapsto -i \end{cases}$$

$$\sigma_7 = \begin{cases} 2^{1/4} \mapsto -2^{1/4} \\ i \mapsto -i \end{cases} \quad \sigma_8 = \begin{cases} 2^{1/4} \mapsto -2^{1/4} i \\ i \mapsto -i \end{cases}$$

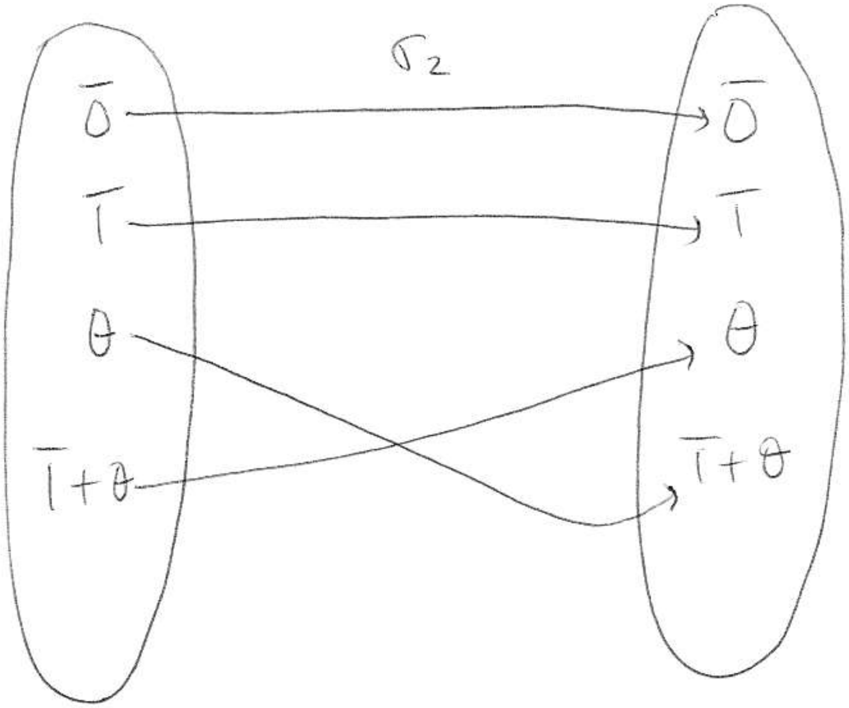
$\mathrm{Gal}(E/\mathbb{Q}) = \{\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_8\}$

ⓒ $x^2+x+\bar{1}$ is irreducible over $\mathbb{Z}_2$ since $\bar{0}^2+\bar{0}+\bar{1}\neq\bar{0}$ and $\bar{1}^2+\bar{1}+\bar{1}\neq\bar{0}$.

Let $\mathbb{F}_4=\mathbb{Z}_2(\theta)=\left\{a+b\theta \;\middle|\; \begin{matrix}\theta^2+\theta+1=0\\ a,b\in\mathbb{Z}_2\end{matrix}\right\}$.

Then, $\mathrm{Gal}(\mathbb{F}_4/\mathbb{Z}_2)=\langle\sigma_2\rangle=\{id,\sigma_2\}$

where $\sigma_2(\alpha)=\alpha^2$ is the Frobenius automorphism.



[Calculation: $(\bar{1}+\theta)^2=\bar{1}+2\theta+\theta^2=\bar{1}+\bar{0}+\bar{1}+\theta=\theta$

$\theta^2=\bar{1}+\theta$]