Fact: Suppose $\sum_{k=1}^{\infty} a_k$ converges absolutely.

Then, $\left|\sum_{k=1}^{\infty} a_k\right| \leq \sum_{k=1}^{\infty} |a_k|$

Proof:

Let $s_n = \sum_{k=1}^{n} a_k$ and $\hat{s}_n = \sum_{k=1}^{n} |a_k|$

Since $\sum_{k=1}^{\infty} a_k$ converges absolutely, both $(s_n)_{n=1}^{\infty}$ and $(\hat{s}_n)_{n=1}^{\infty}$ converge.

Note that $|s_n|$ and $\hat{s}_n$ are real numbers for $n \geq 1$.

By the $\Delta$-inequality,

$$|s_n| = \left|\sum_{k=1}^{n} a_n\right| \leq \sum_{k=1}^{n} |a_n| = \hat{s}_n$$

Thus, $\lim_{n \to \infty} |S_n| \leq \lim_{n \to \infty} \hat{S}_n$,

So, $\left| \sum_{k=1}^{\infty} a_k \right| \leq \sum_{k=1}^{\infty} |a_k|$ $\blacksquare$