

Lemma:

If  $z \neq 1$ , then

$$1 + z + z^2 + \dots + z^k = \frac{1 - z^{k+1}}{1 - z}$$

proof: Suppose  $z \neq 1$ .

$$\text{Let } S_k = 1 + z + z^2 + \dots + z^k.$$

Then,

$$S_k - z \cdot S_k = 1 + z + z^2 + \dots + z^k - z - z^2 - \dots - z^k - z^{k+1}$$

$$= 1 - z^{k+1}$$

$$\text{Thus, } (1 - z) S_k = 1 - z^{k+1}.$$

$$\text{So } S_k = \frac{1 - z^{k+1}}{1 - z}.$$

